On the Coexistence of Cryptocurrency and Fiat Money

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Motivation

- The emergence of Bitcoin has triggered a large wave of public interest in cryptocurrencies
- Cryptocurrency is private money and is costly to produce
  - The new cryptocurrency is produced by "miners" using programming efforts and capital
  - The number of new cryptocurrency that can be produced is decreasing in the total stock
- The number of companies that accept cryptocurrencies as payment methods has been growing

Questions

- Under which conditions can this currency be valued in equilibrium?

  Can it provide price stability?

- Under which conditions can it coexist with government-issued fiat money?

  Would this privately-issued currency be welfare-enhancing?

What I Do

1. A search-theoretic model of an economy with cryptocurrency only
   - Build on Lagos-Wright (2005) Framework
   - Highlights two key attributes: it is private money, and it is costly to produce
     - Supply is endogenous and driven by profit-maximizing miners
     - The marginal production cost depends on aggregate cryptocurrency stock, which subject to currency depreciation/loss

2. Currency competition between cryptocurrency and fiat money
   - Adding government-issued fiat money and multiple decentralized markets
   - Differences: issuers (government vs. miners), supply rules (exogenously vs. endogenously), production costs (costless vs. costly), and degrees of acceptability in decentralized markets

What I Find

- Cryptocurrency-Only Model:
  - Given that the marginal production cost depends on the existing stock of money, the inflation rate must be zero in a stationary monetary equilibrium
    - In sharp contrast to fiat money models and other types of private money economies
- Two-Currency Model:
  - Different from traditional two fiat currencies models, cryptocurrency and fiat money can circulate at the same time and that the rates of return can be different
  - The real values of two currencies are interdependent, and the substitution between them put constraints on government monetary policy
- Policy Implication:
  - If the government tends to overissue fiat money, then banning cryptocurrency would worsen the total welfare

Related Literature: Competing Media of Exchange

- No currency is privately produced
  - Kareken and Wallace (1981); Camera, Crag and Waller, (2004); Velde, Weber and Wright (1999); Zhang (2014)
  - My contribution: Fiat money and cryptocurrency can coexist and that the rates of return on these two assets may not be the same
- At least one of those currencies is privately produced
  - Capital: Lagos and Rocheteau (2008)
  - Crypto/Digital Currency: Fernández-Villaverde and Sanches (2019); Choi and Rocheteau (2020); Zhu and Hendry (2019)
  - My contribution: If cryptocurrency is costly to produce and the new supply is endogenously determined by miners, inflation rate must be zero in equilibrium

Roadmap

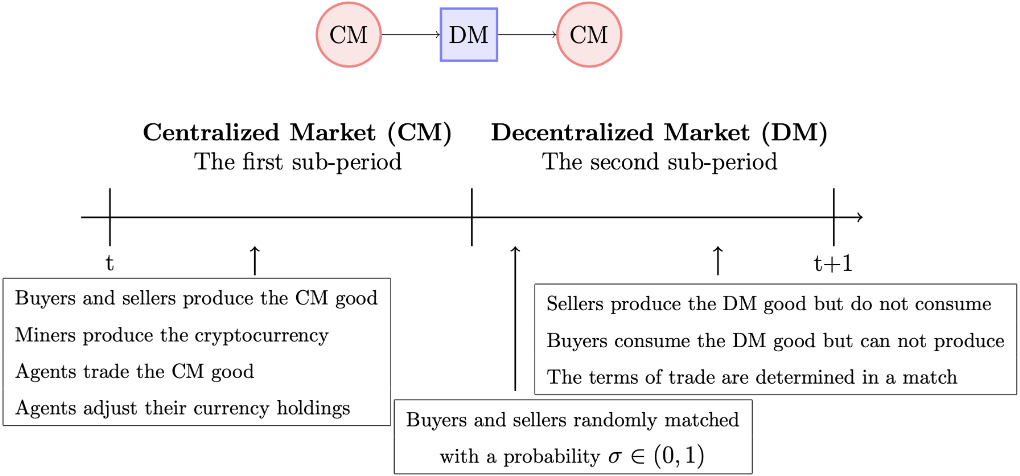
1. Monetary environment of an economy with cryptocurrency only
2. Equilibrium of the cryptocurrency-only model
3. Monetary environment with cryptocurrency and fiat money
4. Equilibrium of the two-currency model
5. Policy implication

A Model of Cryptocurrency Only

- Continuum of ∞-lived agents: *buyers* b, *sellers* s, and *miners* i
- Time is discrete and continues forever, t=0, 1, 2, 3,...
- Each period is divided into two distinct subperiods

1. Centralized market (CM):
    - consume CM goods, produced by b, s
    - i produces cryptocurrency δ
    - everyone trades CM goods x and adjust cryptocurrency holdings m
2. Decentralized market (DM):
    - b, s meet pairwise and at random, and trade DM goods
    - s produces and b consumes
    - terms of trade (q, d)

Timing of Events in a Typical Period



Preferences

- Buyer's: $U^b(x_t^b, q_t) = x_t^b + u(q_t)$
- Seller's: $U^s(x_t^s, q_t) = x_t^s - \omega(q_t)$
- Miner's: $U^i(x_t^i) = x_t^i$

Assumption 1

The functions $u(\cdot)$ and $\omega(\cdot)$ are twice continuously differentiable and satisfy $u(0) = 0,\ u'(0) = \infty,\ \omega(0) = 0,\ \omega'(0) = 0,\ \omega''(0) = 0.$

All consumption goods are non-storable and perfectly divisible

Cryptocurrency

- Supply is endogenous and driven by the production decisions of miners i
- Produce δ_t^i units of new cryptocurrency with the cost $c(\delta_t^i, M_{t-1})$

Assumption 2

The cost function of producing cryptocurrency is increasing, convex, and twice differentiable, and has positive cross derivatives

- Net Circulation:

$$M_t = M_{t-1} + \underbrace{\Delta_t}_{\text{Newly Produced}} - \underbrace{\kappa M_{t-1}}_{\text{Depreciation}}$$

Cryptocurrency is intrinsicallyworthless, perfectly divisible, recognizable, and non-counterfeitable

Problems of Buyers and Sellers

- The Centralized Market maximization problems: $j \in \{b, s\}$

$$W_t^j(m_{t-1}^j) = \max_{x_t^j, m_t^j} \quad x_t^j + V_t^j(m_t^j)$$

$$s.t. \quad x_t^j + p_t m_t^j = p_t(1-\kappa)m_{t-1}^j$$

p_t is the value of cryptocurrency in terms of the CM good

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p_t is the value of cryptocurrency in terms of the CM good

The above CM value functions can be rearranged as:

$$W_t^j(m_{t-1}^j) = p_t(1-\kappa)m_{t-1}^j + \underbrace{\max_{m_t^j \in \mathbb{R}_+} -p_t m_t^j + V_t^j(m_t^j)}_{W_t^j(0)}$$

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$$s.t. \quad x_t^j + p_t m_t^j = p_t(1-\kappa)m_{t-1}^j$$

- The Decentralized Market maximization problems:

$$\text{Buyer:} \quad V_t^b(m_t^b) = \max_{q_t, d_t} \sigma[u(q_t) + \beta W_{t+1}^b(m_t^b - d_t)] + (1-\sigma)[\beta W_{t+1}^b(m_t^b)]$$

$$\text{Seller:} \quad V_t^s(m_t^s) = \quad \sigma[-\omega(q_t) + \beta W_{t+1}^s(m_t^s + d_t)] + (1-\sigma)[\beta W_{t+1}^s(m_t^s)]$$

A buyer b makes a take-it-or-leave-it offer to a seller s if they meet

Decentralized Market

$$\text{Buyer:} \quad V_t^b(m_t^b) = \max_{q_t, d_t} \sigma[u(q_t) + \beta W_{t+1}^b(m_t^b - d_t)] + (1-\sigma)[\beta W_{t+1}^b(m_t^b)]$$

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- A buyer's offer over the terms of trade (q_t, d_t) is given by the problem:

$$\max_{q_t, d_t} \quad u(q_t) + \beta W_{t+1}^b(m_t^b - d_t)$$

$$s.t. \quad -\omega(q_t) + \beta W_{t+1}^s(m_t^s + d_t) \geq \beta W_{t+1}^s(m_t^s)$$

$$d_t \leq m_t^b$$

Terms of Trade

A buyer's problem can be simplified as:

$$\max_{q_t, d_t} \quad u(q_t) - \beta p_{t+1}(1-\kappa)d_t$$

$$s.t. \quad -\omega(q_t) + \beta p_{t+1}(1-\kappa)d_t \geq 0$$

$$d_t \leq m_t^b$$

- Under Assumption 1, $q^* = \text{argmax}\,[u(q_t) - \omega(q_t)]$
- Buyer b chooses (q_t, d_t) such that:

$$q_t(m_t^b) = \begin{cases} q^* & \text{if} \quad m_t^b \geq m^* \\ \hat{q}_t & \text{if} \quad m_t^b < m^* \end{cases}$$

$$d_t(m_t^b) = \begin{cases} m^* & \text{if} \quad m_t^b \geq m^* \\ m_t^b & \text{if} \quad m_t^b < m^* \end{cases}$$

$m_t^* = \frac{\omega(q^*)}{\beta p_{t+1}(1-\kappa)}, \ \hat{q}_t = \omega^{-1}(\beta p_{t+1}(1-\kappa)m_t^b)$

Cryptocurrency Holdings

- The optimal cryptocurrency holdings are given by:

$$W_t^b(m_{t-1}^b) = \max_{m_t^b \in \mathbb{R}_+} - \underbrace{\left(\frac{p_t}{p_{t+1}} - \beta(1-\kappa)\right)p_{t+1}m_t^b}_{\text{cost of carrying money to next period}} + \underbrace{\sigma[u(q_t(m_t^b)) - \omega[u(q_t(m_t^b))]}_{\text{buyer's expected surplus in the DM}}$$

$$W_t^s(m_{t-1}^s) = \max_{m_t^s \in \mathbb{R}_+} - \underbrace{\left(\frac{p_t}{p_{t+1}} - \beta(1-\kappa)\right)p_{t+1}m_t^s}_{\text{cost of carrying money to next period}} + \underbrace{0}_{\text{seller's expected surplus in the DM}}$$

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- Under Assumption 1, the buyer's cryptocurrency holdings m^b are uniquely determined:

$$\frac{p_t}{\beta p_{t+1}(1-\kappa)} - 1 = \sigma[\frac{u' \circ \omega^{-1}(\beta p_{t+1}(1-\kappa)m_t^b)}{\omega' \circ \omega^{-1}(\beta p_{t+1}(1-\kappa)m_t^b)} - 1]$$

Miner's Problem

- In the CM, a typical i chooses x_t^i and δ_t^i

$$\max_{x_t^i, \delta_t^i} \quad x_t^i, \quad s.t. \quad x_t^i \leq p_t \delta_t^i - c(\delta_t^i, M_{t-1})$$

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- Equivalently, $\max_{\delta_t^i \geq 0} \quad p_t \delta_t^i - c(\delta_t^i, M_{t-1})$
- Under Assumption 2, a typical miner i produces:

$$\delta_t^i = c_\delta^{-1}(\max\{p_t, c_\delta(0, M_{t-1})\})$$

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$$\max_{x_t^i, \delta_t^i} \quad x_t^i, \quad s.t. \quad x_t^i \leq p_t \delta_t^i - c(\delta_t^i, M_{t-1})$$

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Aggregate new cryptocurrency Δ_t:

$$\Delta_t = \int_0^1 \delta_t^i di = c_\delta^{-1}(\max\{p_t, c_\delta(0, M_{t-1})\})$$

Equilibrium

- An equilibrium is a set of decision rules in the centralized market, the terms of trade, and sequences of value and aggregate stock of cryptocurrency, such that for all $t \geq 0$:
  - decision rules solve the buyer's and seller's problems in CM and DM
  - the terms of trade in DM maximize the take-it-or-leave-it problem
  - miners choose consumption and production decisions to max their utilities
  - cryptocurrency law of motion is satisfied
  - the cryptocurrency market clear
  - the centralized good market clear

- A stationary equilibrium is an equilibrium in which the real balance of cryptocurrency is constant over time, i.e., $p_t M_t = p_{t+1} M_{t+1} = z^{ss} \ \forall t$

Stationary Equilibrium - Propositions

Under Assumption 1 and Assumption 2:

- There is no stationary monetary equilibrium in which the price of cryptocurrency changes at a constant rate

  There exists a unique stationary monetary equilibrium in which the price of cryptocurrency is constant

  The equilibrium outcomes are characterized by following equations:

$$\frac{1-\beta(1-\kappa)}{\sigma\beta(1-\kappa)} = \left[\frac{u' \circ \omega^{-1}(\beta z^{ss}(1-\kappa))}{\omega' \circ \omega^{-1}(\beta z^{ss}(1-\kappa))} - 1\right]$$

$$f(p^{ss}, M^{ss}) = \kappa M^{ss}$$

$$1 + \frac{1-\beta(1-\kappa)}{\sigma\beta(1-\kappa)} = \frac{u'(q^{ss})}{\omega'(q^{ss})}$$

$$\Delta^{ss} = \kappa M^{ss}$$

Stationary Equilibrium - Propositions

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- Why must the price of cryptocurrency be constant?

  inflationary equilibrium: requires $M_t \uparrow$, but $\partial c_\delta(\delta_t^i, M_{t-1})/\partial M > 0$, production incentive $\downarrow$
  deflationary equilibrium: requires $M_t \downarrow$, but production incentives $\uparrow$ due to high return

Stationary Equilibrium - Propositions

Under Assumption 1 and Assumption 2:

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  There exists a unique stationary monetary equilibrium in which the price of cryptocurrency is constant

- In contrast to fiat money models, e.g., Lagos and Wright (2005)

  supply of money is exogenously given

- In contrast to other types of private money economies, e.g., Fernández-Villaverde and Sanches (2018)

  cost is independent of the existing nominal stock, $c'(0) = 0$

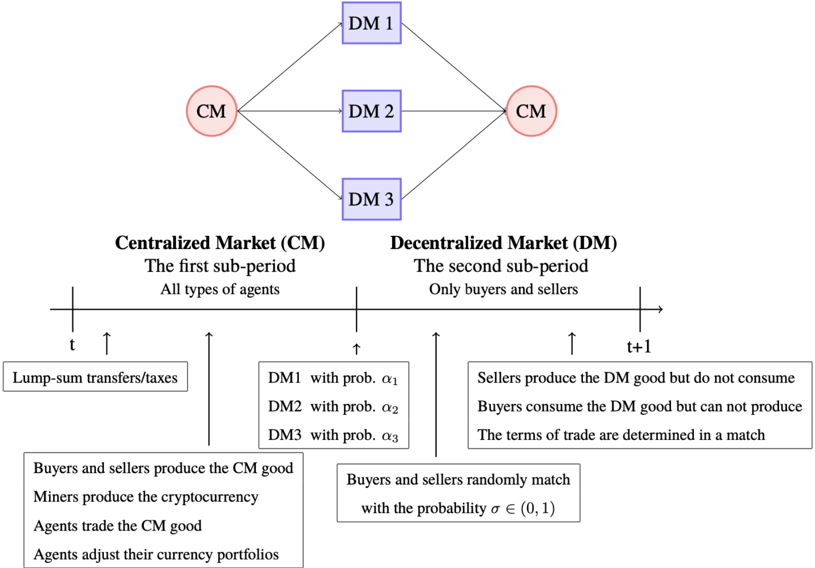
Two-Currency Model

- Cryptocurrency c and Fiat Money m
- Fiat Money
  - Issued by the government, costless to produce
  - Exogenously supplied according to a deterministic growth rule $\gamma \equiv \frac{M^m_{t+1}}{M^m_t}$
  - Changes in the money supply are implemented through lump-sum transfers ($\gamma > 1$) or taxes ($\gamma < 1$) to buyers in the CM

Two-Currency Model

- Continuum of ∞-lived buyers b, sellers s, and miners i
- Each period is divided into two distinct subperiods
  1. CM: i produces δ, agents trade CM goods and adjust currency holdings m^m, m^c
  2. DM: b, s meet pairwise and at random, produce and trade DM goods
     - q_1 in DM1 with α_1, fiat money
     - q_2 in DM2 with α_2, cryptocurrency
     - q_3 in DM3 with α_3, both currencies

Timing of Events in a Typical Period with the Two Currencies



Centralized Market

- Buyer's Problem:

$$W_t^b(\mathbf{m_{t-1}^b}) = \max_{x_t^b, \mathbf{m_t^b}} \; x_t^b + V_t^b(\mathbf{m_t^b})$$
$$s.t. \quad x_t^b + \mathbf{p_t m_t^b} = p_t^m m_{t-1}^{m,b} + (1-\kappa) p_t^c m_{t-1}^{c,b} + T_t$$

- Seller's Problem:

$$W_t^s(\mathbf{m_{t-1}^s}) = \max_{x_t^s, \mathbf{m_t^s}} \; x_t^s + V_t^s(\mathbf{m_t^s})$$
$$s.t. \quad x_t^s + \mathbf{p_t m_t^s} = p_t^m m_{t-1}^{m,s} + (1-\kappa) p_t^c m_{t-1}^{c,s}$$

$$\mathbf{m_{t-1}^b} = (m_{t-1}^{m,b}, m_{t-1}^{c,b}) \in \mathbb{R}_+^2, \; \mathbf{m_{t-1}^s} = (m_{t-1}^{m,s}, m_{t-1}^{c,s}) \in \mathbb{R}_+^2, \mathbf{p}_t = (p_t^m, p_t^c) \in \mathbb{R}_+^2$$

Decentralized Market - Buyer

$$V_t^b(\mathbf{m_t^b}) = \max_{(q_t^1, d_t^{1,m}),(q_t^2, d_t^{2,c}),(q_t^3, \mathbf{d_t^3})} \alpha_1\{\sigma[u(q_t^1) + \beta W_{t+1}^b(m_t^{m,b} - d_t^{1,m}, m_t^{c,b})] + (1-\sigma)\beta W_{t+1}^b(\mathbf{m}_t^b)\}$$

$$+ \alpha_2\{\sigma[u(q_t^2) + \beta W_{t+1}^b(m_t^{m,b}, m_t^{c,b} - d_t^{2,c})] + (1-\sigma)\beta W_{t+1}^b(\mathbf{m}_t^b)\}$$

$$+ \alpha_3\{\sigma[u(q_t^3) + \beta W_{t+1}^b(m_t^{m,b} - d_t^{3,m}, m_t^{c,b} - d_t^{3,c})] + (1-\sigma)\beta W_{t+1}^b(\mathbf{m_t^b})\}$$

Decentralized Market - Seller

$$V_t^s(\mathbf{m_t^s}) = \alpha_1\{\sigma[-\omega(q_t^1) + \beta W_{t+1}^s(m_t^{m,s} + d_t^{1,m}, m_t^{c,s})] + (1-\sigma)\beta W_{t+1}^s(\mathbf{m}_t^s)\}$$

$$+ \alpha_2\{\sigma[-\omega(q_t^2) + \beta W_{t+1}^s(m_t^{m,s}, m_t^{c,s} + d_t^{2,c})] + (1-\sigma)\beta W_{t+1}^s(\mathbf{m}_t^s)\}$$

$$+ \alpha_3\{\sigma[-\omega(q_t^3) + \beta W_{t+1}^s(m_t^{m,s} + d_t^{3,m}, m_t^{c,s} + d_t^{3,c})] + (1-\sigma)\beta W_{t+1}^s(\mathbf{m_t^s})\}$$

The Optimal Currency Portfolio - Buyer

$$W_t^b(m_{t-1}^{m,b}, m_{t-1}^{c,b}) = \max_{m_t^{m,b}, m_t^{c,b} \in \mathbb{R}_+^2} \underbrace{-\left(\frac{p_t^m}{p_{t+1}^m} - \beta\right) p_{t+1}^m m_t^{m,b}}_{\text{cost of holding fiat money}} + \underbrace{-\left(\frac{p_t^c}{p_{t+1}^c} - \beta(1-\kappa)\right) p_{t+1}^c m_t^{c,b}}_{\text{cost of holding cryptocurrency}}$$

$$+ \underbrace{\alpha_1 \sigma[u(q_t^1(m_t^{m,b})) - \beta p_{t+1}^m d_t^{1,m}(m_t^{m,b})]}_{\text{buyer's expected surplus in DM1}}$$

$$+ \underbrace{\alpha_2 \sigma[u(q_t^2(m_t^{c,b})) - \beta(1-\kappa) p_{t+1}^c d_t^{2,c}(m_t^{c,b})]}_{\text{buyer's expected surplus in DM2}}$$

$$+ \underbrace{\alpha_3 \sigma[u(q_t^3(\mathbf{m}_t^b)) - \beta p_{t+1}^m d_t^{3,m}(\mathbf{m}_t^b) - \beta(1-\kappa) p_{t+1}^c d_t^{3,c}(\mathbf{m}_t^b)]}_{\text{buyer's expected surplus in DM3}}$$

The Optimal Currency Portfolio - Seller

$$W_t^s(m_{t-1}^{m,s}, m_{t-1}^{c,s}) = \max_{m_t^{m,s}, m_t^{c,s} \in \mathbb{R}_+^2} \underbrace{-(\frac{p_t^m}{p_{t+1}^m} - \beta)p_{t+1}^m m_t^{m,s}}_{\text{cost of holding fiat money}} + \underbrace{-(\frac{p_t^c}{p_{t+1}^c} - \beta(1-\kappa))p_{t+1}^c m_t^{c,s}}_{\text{cost of holding cryptocurrency}}$$

$$+ \underbrace{0}_{\text{expected surplus in DM 1}} + \underbrace{0}_{\text{expected surplus in DM 2}} + \underbrace{0}_{\text{expected surplus in DM 3}}$$

Coexistence - Proposition

- Under Assumptions 1 and 2, there exists a stationary equilibrium in which
  - both cryptocurrency and fiat money are valued
  - the price of cryptocurrency is constant
  - the price of fiat money changes at a constant rate s.t. $p^m_{t+1} = \frac{1}{\gamma} p^m_t$

  so long as $\beta < \gamma < \bar{\gamma} \equiv \beta \alpha_1 \sigma L(0) + \frac{\alpha_3}{\alpha_2 + \alpha_3} (\frac{1}{1-\kappa} - \beta) + \beta$ and $0 < \hat{\mu} \equiv (\alpha_2 \sigma L(0) + 1) \beta (1 - \kappa) - 1$.

- Cryptocurrency and fiat money can coexist in equilibrium regardless of their rates of return (different degrees of acceptability in decentralized markets)

Coexistence Equilibrium Outcomes

$$\frac{\gamma-\beta}{\sigma\beta} = \alpha_1[\frac{u'\circ\omega^{-1}(\beta\frac{z_m}{\gamma})}{\omega'\circ\omega^{-1}(\beta\frac{z_m}{\gamma})}-1]+\alpha_3[\frac{u'\circ\omega^{-1}(\beta(\frac{z_m}{\gamma}+z_c(1-\kappa)))}{\omega'\circ\omega^{-1}(\beta(\frac{z_m}{\gamma}+z_c(1-\kappa)))}-1]$$

$$\frac{1-\beta(1-\kappa)}{\sigma\beta(1-\kappa)} = \alpha_2[\frac{u'\circ\omega^{-1}(\beta z_c(1-\kappa))}{\omega'\circ\omega^{-1}(\beta z_c(1-\kappa))}-1]+\alpha_3[\frac{u'\circ\omega^{-1}(\beta(\frac{z_m}{\gamma}+z_c(1-\kappa)))}{\omega'\circ\omega^{-1}(\beta(\frac{z_m}{\gamma}+z_c(1-\kappa)))}-1]$$

$$c_\delta^{-1}(p_c^{ss}) = \kappa M_c^{ss}$$

$$q_1^{ss} = \omega^{-1}(\beta\frac{z_m}{\gamma})$$

$$q_2^{ss} = \omega^{-1}(\beta z_c(1-\kappa))$$

$$q_3^{ss} = \omega^{-1}(\beta\frac{z_m}{\gamma}+\beta z_c(1-\kappa))$$

$$\Delta^{ss} = \kappa M_c^{ss}$$

Comparative Statics

Buyers can make offers on any arbitrary mix of the two currencies in DM3

- As the fiat money inflates ($\gamma \uparrow$), it becomes more costly to use

  Demand less for fiat money and instead substitute into cryptocurrency, $\downarrow$ the real value of fiat money and $\uparrow$ that of cryptocurrency

Comparative Statics

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  Demand less for fiat money and instead substitute into cryptocurrency, $\downarrow$ the real value of fiat money and $\uparrow$ that of cryptocurrency

- If cryptocurrency is lost at a higher rate ($\kappa\ \uparrow$), or if the marginal cost of producing cryptocurrency diminishes ($c_\delta(\delta, M^c)\ \downarrow$), the cost of carrying cryptocurrency increases

  Demand less for cryptocurrency and more for fiat money in decentralized meetings, which decreases the real value of cryptocurrency and increases that of fiat money

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- The market size $\alpha_3 \neq 0$ is necessary to put constraints on government monetary policy

  $\alpha_3 = 0$: two completely segmented decentralized markets. Government monetary policy has no effect on cryptocurrency use.

The Laffer Curve in Fiat Money Model Parameters

Laffer curve for the inflation tax: $z_m(\gamma)(1-\frac{1}{\gamma})$

Set $u(q)=\frac{q^{1-a}}{1-a}$ with $a<1$ and $\omega(q)=q$

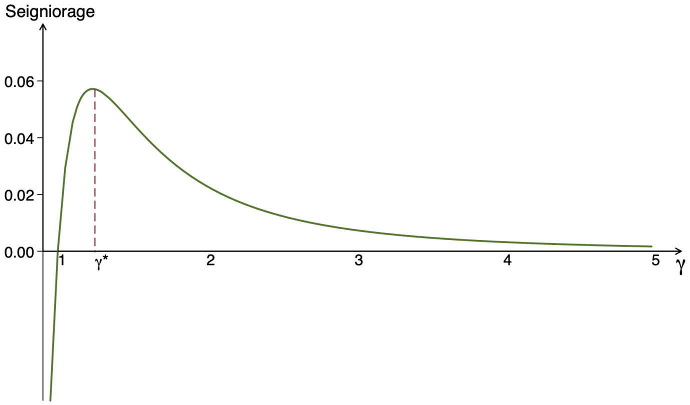

Figure 1. The Laffer Curve in Fiat Money Model

The Laffer Curve in Two-Currency Model

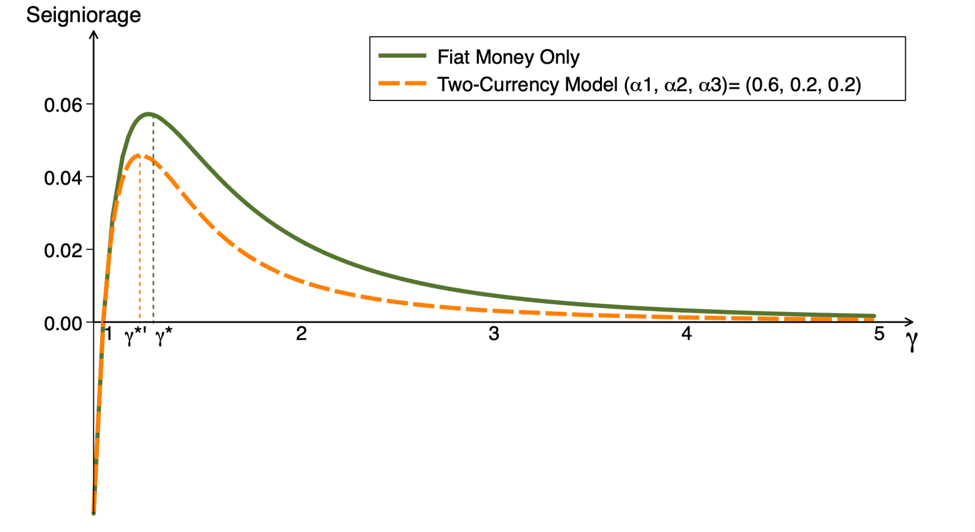

Figure 2. Fiat Money Only vs. With Cryptocurrency

The Laffer Curve in Two-Currency Model

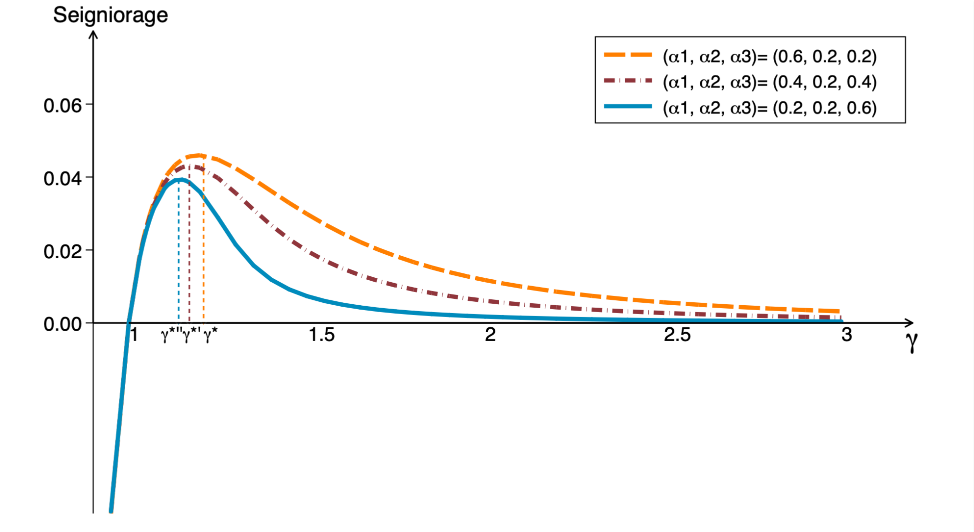

Figure 3. Effects of Changing $\alpha_1, \alpha_2, \alpha_3$

Policy Implication

Should the government ban cryptocurrency?

- Cryptocurrency is costly to produce, banning it avoids the resource cost on production. Agents in DM2 are only allowed to trade using cryptocurrency.
- Worsen the welfare: if the government tends to over-issue money
- Welfare-enhancing: if the government can maintain sufficiently low inflation and the market size of the DM2 is small

As cryptocurrency is consistent with zero inflation in a stationary monetary equilibrium, the government that tends to use the inflation tax would have a strong incentive to ban cryptocurrency

Summary

This paper uses a search-theoretical model to study conditions

- Under which cryptocurrency can be valued in equilibrium
- Under which it can coexist with fiat money

Cryptocurrency-only model

- The marginal production cost depends on the existing stock of money
- The inflation rate must be zero in a stationary monetary equilibrium

Two-currency model

- Fiat money and cryptocurrency can coexist with different rates of return
- Competing with cryptocurrency restricts over-issue currency problem of fiat money

Thank You!

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CM Value Function Back

The CM value functions can be rearranged as:

$$W_t^j(m_{t-1}^j) = p_t(1-\kappa)m_{t-1}^j + W_t^j(0), \quad j \in \{b, s\}$$

Terms of Trade Back

- $A_{DM,t}$: the total value of assets that are used for trading in the DM and period t

- If $A_{DM,t} \geq \omega(q^*)$

$$(q_t^{DM}, d_t^{DM}) = \begin{cases} q_t^1 = q^*, & d_t^{1,m} = m_t^{m*} = \frac{\omega(q^*)}{\beta p_{t+1}^m} \\ q_t^2 = q^*, & d_t^{2,c} = m_t^{c*} = \frac{\omega(q^*)}{\beta p_{t+1}^c (1-\kappa)} \\ q_t^3 = q^*, & (d_t^{3,m}, d_t^{3,c}) = (\hat{m}_t^{m,b}, \hat{m}_t^{c,b}) \\ & \quad s.t. \ \omega(q^*) = \beta(p_{t+1}^m \hat{m}_t^{m,b} + p_{t+1}^c (1-\kappa)\hat{m}_t^{c,b}) \end{cases}$$

- If $A_{DM,t} < \omega(q^*)$

$$(q_t^{DM}, d_t^{DM}) = \begin{cases} q_t^1 = \hat{q}_t^1 = \omega^{-1}(A_{1,t}), & d_t^{1,m} = m_t^{m,b} \\ q_t^2 = \hat{q}_t^2 = \omega^{-1}(A_{2,t}), & d_t^{2,c} = m_t^{c,b} \\ q_t^3 = \hat{q}_t^3 = \omega^{-1}(A_{3,t}), & (d_t^{3,m}, d_t^{3,c}) = (m_t^{m,b}, m_t^{c,b}) \end{cases}$$

Parameter Values in Plotting Laffer Curves

Back

	Fiat Money Only	Two-Currency Model		Changing Acceptability		
	(1)	(2)	(3)	(4)	(5)	(6)
a	0.5	0.5	0.5	0.5	0.5	0.5
β	0.95	0.95	0.95	0.95	0.95	0.95
σ	0.5	0.5	0.5	0.5	0.5	0.5
κ	N/A	N/A	0.2	0.2	0.2	0.2
α_1	1	1	0.6	0.6	0.4	0.2
α_2	0	0	0.2	0.2	0.2	0.2
α_3	0	0	0.2	0.2	0.4	0.6